1. Present and analyze
   (a) a random access machine,
   (b) a 2-tape Turing machine, and
   (c) a 1-tape Turing machine

   which computes the function $1^n \mapsto 1^{2n}$ ($n \in \mathbb{N}$). (That is, the machine reads a sequence of characters '1' from its input tape, and writes a sequence of twice that many ones on its output tape.)

2. Sketch a nondeterministic Turing machine that recognizes $n$-bit long binary palindromes using time about $n$ and space about $\frac{3}{2}n$, only.

3. Present a deterministic two-tape Turing machine which receives two binary numbers of length $n \geq 1$, separated by a '+' sign, and computes their sum on the result tape. Analyze the time complexity of the machine.

4. Let $M$ be a random access machine, whose time complexity according to the logarithmic-cost model is $T(n)$. At the lecture we sketched a proof showing that $M$ can be simulated by a Turing machine $M'$ whose time complexity is $O(T(n)^3)$. Sketch how TM $M'$ would simulate the RAM instruction `store *2` in time $O(T(n))$. Assume that at the start the contents of AC is stored, in binary, on the auxiliary tape of $M'$.

5. Give a yes/no answer, with a short justification, to the following questions:
   (a) If some problem of class $\mathcal{NP}$ is solvable in polynomial time, are then (i) all problems of class $\mathcal{NP}$, or (ii) all NP-complete problems solvable in polynomial time?
   (b) If some NP-complete problem is solvable in polynomial time, are then (i) all problems of class $\mathcal{NP}$, or (ii) all NP-complete problems solvable in polynomial time?
   (c) If some problem of class $\mathcal{NP}$ requires super-polynomial time, do then (i) all problems of class $\mathcal{NP}$, or (ii) all NP-complete problems require super-polynomial time?
   (d) If some NP-complete problem requires super-polynomial time, do then (i) all problems of class $\mathcal{NP}$, or (ii) all NP-complete problems require super-polynomial time?